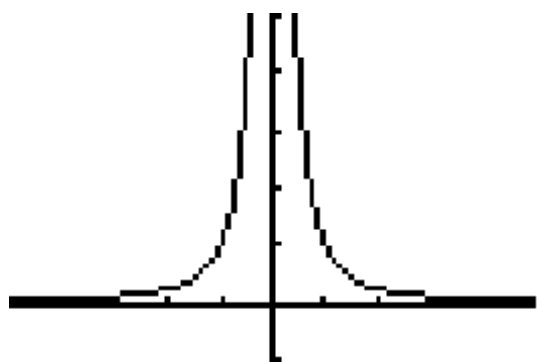


2-1 Rates of Change and Limits

Learning Targets

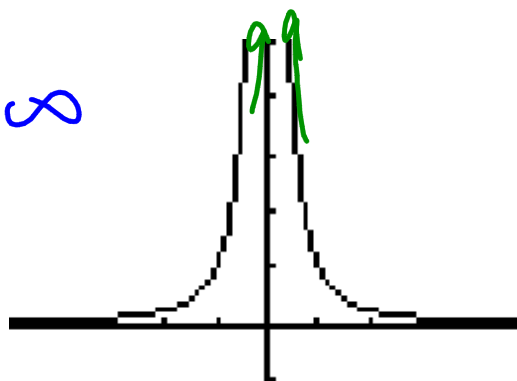
- I can find the value of a limit by looking at the graph of a function.
- I can calculate limits algebraically.
- I can calculate one and two sided limits.
- I can determine when a limit exists.

Graph $f(x) = \frac{1}{x^2}$

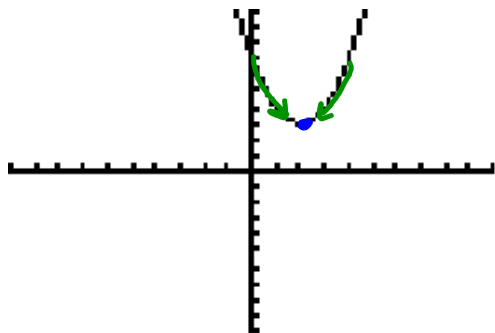


Limits

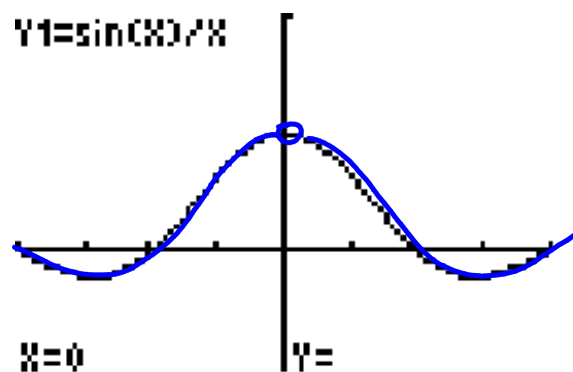
Find $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$



Find $\lim_{x \rightarrow 2} (x - 2)^2 + 3 = 3$



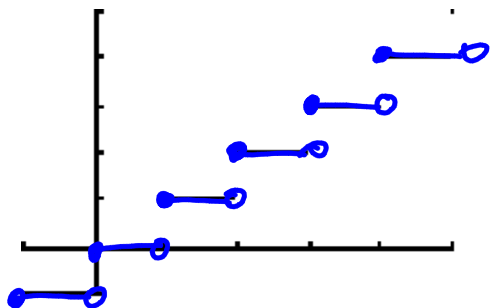
Find $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$



Find $\lim_{x \rightarrow 1} [x] = \text{DNE}$

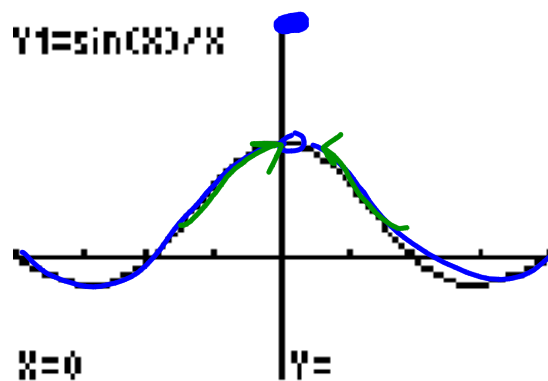
$$\lim_{x \rightarrow 1^+} [x] = 1$$

$$\lim_{x \rightarrow 1^-} [x] = 0$$



$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Find $\lim_{x \rightarrow 0} f(x) = 1$

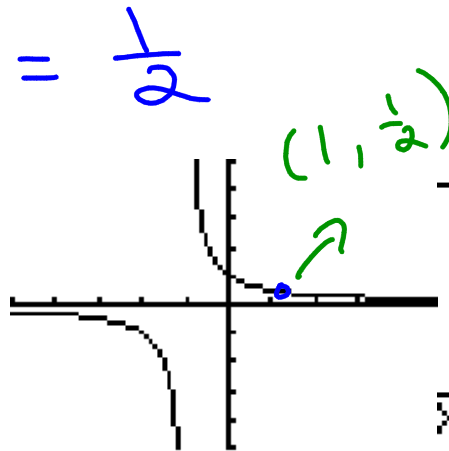


Ex1. Evaluate the limit (graphically)

$$1. \lim_{x \rightarrow 1} \frac{x-1}{x^2-1} = \frac{1}{2}$$

```

Plot1 Plot2 Plot3
Y1=(X-1)/(X^2-1)
Y2=
Y3=
Y4=
Y5=
Y6=
    
```



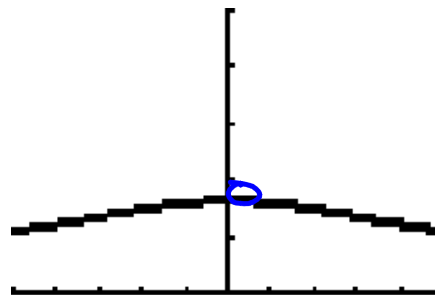
X	Y1
1	ERROR
1.1	.47619
1.01	.49751
.9	.52632
.99	.50251

X=

$$2. \lim_{t \rightarrow 0} \frac{\sqrt{t^2+9}-3}{t^2} = .1\bar{6} = \frac{1}{6}$$

```

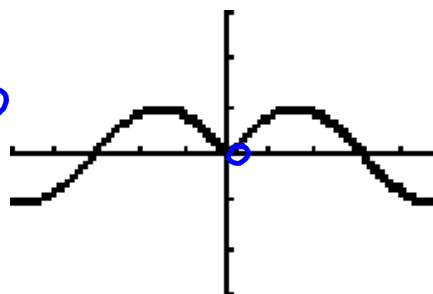
Plot1 Plot2 Plot3
Y1=(X-1)/(X^2-1)
Y2=sin(X)/|X|
Y3=(sqrt(X^2+9)-3)/X^2
Y4=
Y5=
Y6=
    
```



X	Y3
0	ERROR
.1	.16662
.01	.16667
.001	.16667
-.1	.16662
-.01	.16667
-.001	.16667

X= -.001

$$3. \lim_{x \rightarrow 0} \frac{x \sin x}{|x|} = 0$$



X	Y2
0	ERROR
.1	.09983
.01	.01
.001	1E-3
-.1	.09983
-.01	.01
-.001	1E-3

X= -.001

Ex2. Evaluate the Limits (algebraically)

$$1. \lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 4} = \frac{2^2 - 7 \cdot 2 + 10}{2^2 - 4} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{(x-5)(\cancel{x-2})}{(x+2)(\cancel{x-2})} = \lim_{x \rightarrow 2} \frac{x-5}{x+2} = \left(\frac{-3}{4} \right)$$

X	Y1	Y2
2	ERROR	-.75

X=

$$2. \lim_{x \rightarrow 0} \frac{(x+3)^3 - 27}{x}$$

Handwritten solution for the limit problem:

$$\lim_{x \rightarrow 0} \frac{(x+3)^3 - 27}{x} = \frac{0}{0}$$

$$\frac{(x+3)(x+3)(x+3)}{(x^2+6x+9)(x+3)}$$

$$x^3 + 3x^2 + 6x^2 + 18x + 9x + 27$$

$$\frac{x^3 + 9x^2 + 27x + 27}{x}$$

$$= \lim_{x \rightarrow 0} (x^2 + 9x + 27)$$

$$= 0 + 0 + 27$$

$$= 27$$

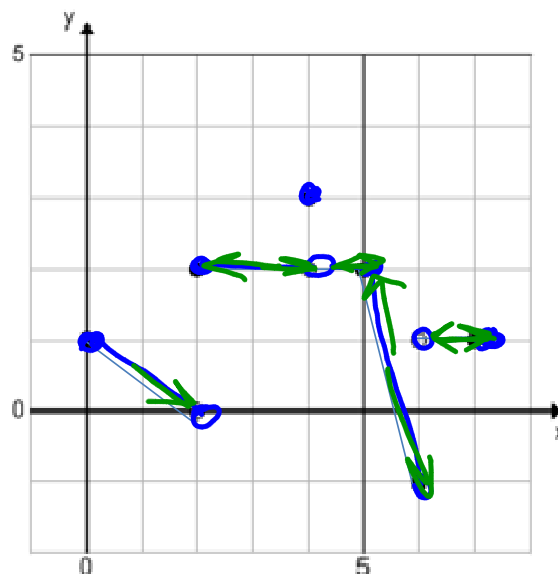
$$3. \lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{\frac{x}{1}}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = 1 \cdot 1 = 1$$

Ex3. Use the graph to evaluate

- a. $\lim_{x \rightarrow 4} f(x)$ 2
- b. $\lim_{x \rightarrow 2^-} f(x)$ 0
- c. $\lim_{x \rightarrow 2^+} f(x)$ 2
- d. $\lim_{x \rightarrow 2} f(x)$ DNE
- e. $\lim_{x \rightarrow 5} f(x)$ 2
- f. $\lim_{x \rightarrow 7} f(x)$ DNE
- g. $\lim_{x \rightarrow 7^-} f(x)$ 1
- h. $\lim_{x \rightarrow 6} f(x)$ DNE



Ex4. Find the limit.

1. $\lim_{x \rightarrow 0} \frac{2 \cos x \sin x}{x}$

2. $\lim_{x \rightarrow 0} \frac{x^2 \cos x + x \sin x + 4x^2}{x^2}$

Homework

p. 66 #7, 9, 11, 15, 16, 19, 20, 24, 27, 28,
31-36, 38, 40, 42, 44, 49